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The nonlinear pulse propagation in an optical fibers with varying parameters is investigated. The capture of moving in the frequency domain femtosecond colored soliton by a dispersive trap formed in an amplifying fiber makes it possible to accumulate an additional energy and to reduce significantly the soliton pulse duration. Nonlinear dynamics of the chirped soliton pulses in the dispersion managed systems is also investigated. The methodology developed does provide a systematic way to generate infinite "ocean" of the chirped soliton solutions of the nonlinear Schrödinger equation (NSE) with varying coefficients.

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I. INTRODUCTION

In 1973 Hasegawa and Tappert [1] showed theoretically that an optical pulse in a dielectric fibers forms an envelope solitons, and in 1980 Mollenauer, Stolen and Gordon [2] demonstrated the effect experimentally. This discovery is significant in its application to optical communications. Today the optical soliton is regarded as an important alternative for the next generation of high speed telecommunication systems.

The theory of NSE solitons was developed for the first time in 1971 by Zakharov and Shabad [3]. The concept of the soliton involves a large number of interesting problems in applied mathematics since it is an exact analytical solution of a nonlinear partial differential equations. The theory of optical solitons described by the nonlinear Schrödinger equation has produced perfect agreement between theory and experiment [4].

In this paper we present mathematical description of solitary waves propagation in a nonlinear dispersive medium with varying parameters.

The soliton spectral tunneling effect was theoretically predicted in [5]. This is characterized in the spectral domain by the passage of a femtosecond soliton through a potential barrier-like spectral inhomogeneity of the group velocity dispersion (GVD), including the forbidden band of a positive GVD. It is interesting to draw an analogy with quantum mechanics where the solitons are considered to exhibit particle-like behavior. The soliton spectral tunneling effect also can be considered as an example of the dynamic dispersion soliton management technique. In the first part of the paper we will concentrate on the problem of femtosecond solitons amplification. We will show that spectral inhomogeneity of GVD allows one to capture a soliton in a sort of spectral trap and to accumulate an additional energy during the process of the soliton amplification. In the second part we will consider the problem of the short soliton pulse propagation in the nonlinear fiber with static non-uniform inhomogeneity of GVD. The methodology developed does provide a systematic way to generate infinite "ocean" of the chirped soliton solutions of NSE model with varying coefficients.

II. FEMTOSECOND SOLITON AMPLIFICATION

It is well known that due to the Raman self-scattering effect [6] (called soliton self-frequency shift [7]) the central femtosecond soliton frequency shifts to the red spectral region and so-called colored solitons are generated. This effect decreases significantly the efficiency of resonant amplification of femtosecond solitons. The mathematical model we consider based on the modified NSE including the effects of molecular vibrations and soliton amplification processes (see details in [8]):

$$i\frac{\partial\psi}{\partial z}=\frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2}+i\sigma\frac{\partial^3\Psi}{\partial\tau^3}+(1-\beta)|\psi|^2\psi+\beta Q\psi+\frac{G}{2}P \tag{1}$$

$$\mu^2\frac{\partial^2 Q}{\partial t^2}+2\mu\delta\frac{\partial Q}{\partial t}+Q=|\psi|^2,\quad \text{and,}\quad \gamma_a\frac{\partial P}{\partial\tau}+P(1+i\gamma_a\Delta\Omega)=i\psi, \tag{2}$$

As numerical experiments showed the GVD inhomogeneity as a potential well allows one to capture a soliton in a sort of spectral trap. Figure 1 shows the nonlinear dynamics of the soliton spectral trapping effect in the spectral domain. As soliton approaches the well, it does not slow down but speeds up, and then, after it has got into the well, the soliton is trapped. There exists a long time of soliton trapping in internal region of the well .This effect opens a controlled possibility to increase the energy of a soliton. As follows from our computer simulations the capture of moving in the frequency space femtosecond colored soliton by a dispersive trap formed in an amplifying optical fiber makes it possible to accumulate an additional energy in the soliton dispersive trap and to reduce significantly the soliton pulse duration.

III. DISPERSION MANAGEMENT: CHIRPED SOLITONS

Let us consider the propagation of a nonlinear pulse in the anomalous (or normal) group velocity dispersion fiber of length Z_1. The complex amplitude q of the light wave in a fiber with variable parameters $D_2(Z)$, $N_2(Z)$ and $\Gamma(Z)$ is described by the nonlinear Schrodinger equation

$$i\frac{\partial q}{\partial Z}+\frac{1}{2}D_2(Z)\frac{\partial^2 q}{\partial T^2}+N_2(Z)\mid q\mid^2 q=i\Gamma(Z)q \tag{3}$$

Theorem 1. Consider the NSE (3) with varying dispersion, nonlinearity and gain. Suppose that Wronskian $W[N_2,D_2]$ of the functions $N_2(Z)$ and $D_2(Z)$ is nonvanishing, thus two functions $N_2(Z)$ and $D_2(Z)$ are linearly independent. There are then infinite number of solutions of Eq. (3) in the form of Eq.4

$$q(Z,T)=\sqrt{\frac{D_2(Z)}{N_2(Z)}}\;P(Z)\;Q\left[P(Z)\cdot T\right]\;\exp\left[i\frac{P(Z)}{2}\;T^2+i\int\limits_0^Z K(Z^{'})dZ^{'}\right] \tag{4}$$

where function Q describes fundamental functional form of bright $Q=\mathrm{sech}(P(Z)T)$ or dark $Q=\mathrm{th}(P(Z)T)$ NSE solitons and the real functions $P(Z)$, $D_2(Z)$, $N_2(Z)$ and $\Gamma(Z)$ are determined by the following nonlinear system of equations :

$$\frac{1}{P^2(Z)}\frac{\partial P(Z)}{\partial Z}+D_2(Z)=0\;;\;-\frac{1}{2}D_2(Z)P(Z)+\frac{W\left[N_2(Z),D_2(Z)\right]}{2D_2(Z)N_2(Z)}=\Gamma(Z) \tag{5}$$

Theorem 2. Consider the NSE (3) with varying dispersion, nonlinearity and gain. Suppose that Wronskian $W[N_2,D_2]$ of the functions $N_2(Z)$ and $D_2(Z)$ is vanishing, thus two functions $N_2(Z)$ and $D_2(Z)$ are linearly dependent. There are then infinite number of solutions of Eq. (3) of the following form Eq. 6

$$q(Z,T)=C\;P(Z)\;Q\left[P(Z)\cdot T\right]\;\exp\left[i\frac{P(Z)}{2}\;T^2+i\int\limits_0^Z K(Z^{'})dZ^{'}\right] \tag{6}$$

where function Q describes the fundamental form of bright (or dark) NSE soliton and the real functions $P(Z)$, $D_2(Z)$, $N_2(Z)$ and $\Gamma(Z)$ are determined by the following nonlinear system of equations :

$$D_2(Z)=-\frac{1}{P^2(Z)}\frac{\partial P(Z)}{\partial Z}\;;\;\Gamma(Z)=\frac{1}{2}\frac{1}{P}\frac{\partial P(Z)}{\partial Z}\;;\;N_2(Z)=D_2(Z)/C^2 \tag{7}$$

The function P(Z) is required only to be once-differentiable, but otherwise arbitrary function, there is no restrictions.

To prove Theorems 1 and 2 we first construct a stationary localized solution of Eq. (3) by introducing Kumar-Hasegawa's quasi-soliton concept [9–11] through

$$
q(Z,T) = \sqrt{\frac{D_2(Z)}{N_2(Z)}}\; P(Z)\; Q\left[P(Z)\cdot T\right]\; \exp\left[i\frac{P(Z)}{2}\; T^2 + i\int\limits_0^Z K(Z^{'})dZ^{'} \right] \tag{8}
$$

where D_2(Z), N_2(Z), $P(Z)$ and $K(Z)$ are the real functions of Z. Substituting expression (8) into Eq. (3) and separating real and imaginary parts we obtain the system of two equations

$$
\frac{1}{2}\mathrm{sign}(D_2)\frac{\partial^2 Q}{\partial S^2} + Q^3 + \left(E - \frac{S^2}{2}\cdot \Omega^2(Z) \right)\; Q = 0 \tag{9}
$$

$$
\frac{\partial P}{\partial Z}Q + P\frac{\partial Q}{\partial S}\frac{\partial S}{\partial Z} + \frac{1}{2}\frac{1}{D_2(Z)}\frac{\partial D_2}{\partial Z}PQ - \frac{1}{2}\frac{1}{N_2(Z)}\frac{\partial N_2}{\partial Z}PQ + \frac{1}{2}D_2P^2Q + D_2P^2T\frac{\partial Q}{\partial S}\frac{\partial S}{\partial T} = \Gamma PQ \tag{10}
$$

Where

$$
S(Z,T) = P(Z)T\; ;\; \frac{\partial S}{\partial Z} = T\frac{\partial P}{\partial Z}\; ;\; \frac{\partial S}{\partial T} = P(Z) \tag{11}
$$

In Eq. (9) the parameters E and Ω are 'the energy' and 'frequency' of ordinary quantum mechanical harmonic ocsillator

$$
\Omega^2(Z) = \frac{D_2^{-1}(Z)}{P^2(Z)}\left(\frac{1}{P^2(Z)}\frac{\partial P}{\partial Z} + D_2(Z) \right)\; ;\; E(Z) = -K(Z)/P^2(Z)/D_2(Z) \tag{12}
$$

Eq. (9) represents the nonlinear Schrodinger equation for the harmonic ocsillator. As must be in the case of Hamiltonian system Eq. (9) may be written in the form

$$
\frac{\delta H}{\delta Q^*} = 0 \tag{13}
$$

$$
H = \int \left[\frac{1}{2}\mathrm{sign}(D_2)\left|\frac{\partial Q}{\partial X}\right|^2 + \frac{1}{2}\alpha\left|Q\right|^4 + \left(E - \frac{X^2}{2}\cdot\Omega^2(Z) \right)\; \left|Q\right|^2 \right] dX \tag{14}
$$

The derivative in (13) is functional derivative. For the first time this equation was solved numerically by Kumar and Hasegawa in [9] and gave rise a new concept of quasi-solitons [10,11]. Now we make the important assumption about the solution of Eq. (9).

Let us consider the complete nonlinear regime when Eq. (9) represents the ideal NLS equation, i.e. we will allow $\Omega(Z) \equiv 0$, then from (12) follows that

$$
\frac{1}{P^2(Z)}\frac{\partial P(Z)}{\partial Z} + D_2(Z) = 0 \tag{15}
$$

We now look for a solution of Eq. (10) which satisfies the condition (15). Substituting the expression (15) and relations (11) into Eq. (10) we obtain

$$
-\frac{1}{2}D_2(Z)P(Z) + \frac{1}{2}\frac{1}{D_2(Z)}\frac{\partial D_2(Z)}{\partial Z} - \frac{1}{2}\frac{1}{N_2(Z)}\frac{\partial N_2(Z)}{\partial Z} = \Gamma(Z) \tag{16}
$$

Using notation

$$
W\left\{N_2, D_2\right\} = N_2\frac{\partial D_2(Z)}{\partial Z} - D_2\frac{\partial N_2(Z)}{\partial Z}
$$

one can obtain the soliton solution of Eq. 3 in the form of the chirped solitons Eqs. 4-5 and Eqs. 6-7.. Consequently, we have found the infinite "ocean" of solutions. The methodology developed does provide a systematic way of new and new chirped soliton solutions generation.

IV. DIFFERENT REGIMES OF SOLITON MANAGEMENT

Lemma 1: Soliton GVD management. Consider the NSE (3) with constant nonlinear coefficient $N_2 = const$ and with varying along Z-coordinate GVD parameter. Suppose that dispersion management function is known arbitrary analytical function :$D_2(Z)=\Phi(Z)$. The function $\Phi(Z)$ is required only to be once-differentiable and once integrable, but otherwise arbitrary function, there is no restrictions. There are then infinite number of solutions of Eq. (3) of the form of the chirped dispersion managed dark and bright solitons Eq. 4, where the main functions P and Γ are given by

$$D_2(Z) = \Phi(Z) \ ; \ P(Z) = -\frac{1}{\left[C - \int \Phi(Z) dZ\right]} \tag{17}$$

$$\Gamma(Z) = \frac{1}{2} \frac{\Phi(Z)}{\left[C - \int \Phi(Z) dZ\right]} + \frac{1}{2} \frac{1}{\Phi(Z)} \frac{\partial \Phi(Z)}{\partial Z} \tag{18}$$

Lemma 2: Soliton intensity management. Consider the NSE (3) with constant nonlinear coefficient $N_2 = const$ and with varying along Z-coordinate the dispersion and gain. Suppose that intensity of the soliton pulse is determined by the known management function: $D_2(Z)P^2(Z)=\Theta(Z)$,where the function $\Theta(Z)$ is required only to be once-differentiable and once integrable, but otherwise arbitrary function, there is no restrictions. There are then infinite number of solutions of Eq. (3) of the form of the chirped dispersion managed dark and bright solitons Eq. 4 with parameters given by

$$D_2(Z)P^2(Z) = \Theta(Z) \ ; \ P(Z) = -\int \Theta(Z) dZ + C \ ; \ D_2(Z) = \frac{\Theta(Z)}{\left[C - \int \Theta(Z) dZ\right]^2}$$

$$\Gamma(Z) = \frac{1}{2} \frac{\Theta(Z)}{\left[C - \int \Theta(Z) dZ\right]} + \frac{1}{2} \frac{1}{\Theta(Z)} \frac{\partial \Theta(Z)}{\partial Z} \tag{19}$$

Lemma 3: Soliton pulse duration management: optimal soliton compression. Consider the NSE (3) with constant nonlinear coefficient $N_2 = const$ and with varying along Z-coordinate the dispersion and gain coefficients. Suppose that pulse duration of a soliton is determined by the known analytical function: $P(Z)=\Upsilon(Z)$, where the function $\Upsilon(Z)$ is required only to be two-differentiable , but otherwise arbitrary function, there is no restrictions. There are then infinite number of solutions of Eq. (3) of the form of the chirped dispersion managed dark and bright solitons Eq. 4 with the main parameters given by

$$D_2(Z) = -\frac{1}{\Upsilon^2(Z)} \frac{\partial \Upsilon(Z)}{\partial Z} \ ; \Gamma(Z) = \frac{1}{2} \left(\frac{\partial \Upsilon(Z)}{\partial Z} \right)^{-1} \frac{\partial}{\partial Z} \left(\frac{1}{\Upsilon(Z)} \frac{\partial \Upsilon(Z)}{\partial Z} \right) \tag{20}$$

Lemma 4: Soliton amplification management: optimal soliton compression. Consider the NSE (3) with constant nonlinear coefficient $N_2 = const$ and with varying along Z-coordinate the dispersion and gain coefficients. Suppose that the gain coefficient is determined by the known control function: $\Gamma(Z)=\Lambda(Z)$, where the function $\Lambda(Z)$ is required only to be once integrable , but otherwise arbitrary function, there is no restrictions. There are then infinite number of solutions of Eq. (3) of the form of the chirped dispersion managed dark and bright solitons of the Eq. 4 where

$$|P(Z)| = \exp\left[\int \exp\left(\int 2\Lambda(Z'') dZ'' \right) dZ' \right] \tag{21}$$

$$|D_2(Z)| = \frac{\exp\left(\int 2\Lambda(Z) dZ\right)}{\exp\left[\int \exp\left(\int 2\Lambda(Z'') dZ''\right) dZ'\right]} \tag{22}$$

Lemma 5: Combined dispersion and nonlinear soliton management. Consider the NSE (3) with varying nonlinear coefficient $N_2(Z)$ and with varying along Z-coordinate the dispersion and gain coefficients too. Suppose that Wronskian $W[N_2,D_2]$ is vanishing, or that the functions $N_2(Z)$ and $D_2(Z)$ are linearly dependent. Suppose also that the function $D_2(Z)$ is determined by the initial control function $D_2(Z)=\Xi(Z)$,where the function $\Xi(Z)$ is required only to be once integrable, but otherwise arbitrary function, there is no restrictions. There are then infinite number of solutions of Eq. (3) of the form of the chirped dispersion managed dark and bright solitons of the Eq. 6 where

$$P(Z) = -\frac{1}{[C - \int \Xi(Z)dZ]} \ ; \ N_2(Z) = D_2(Z)/C^2 \tag{23}$$

$$\Gamma(Z) = \frac{1}{2}\frac{\Xi(Z)}{[C - \int \Xi(Z)dZ]} \tag{24}$$

The analytical solutions for the different regimes of the main soliton parameters management (intensity, pulse duration, amplification or absorption) in the case of W[N_2,D_2]=0 can be obtained by using theorem 2.

Let us consider some examples. The case of $\Gamma(Z) \equiv 0$ and N_2(Z)=N_2(0) corresponds to the problem of ideal GVD soliton management. The soliton solution in this case is:

$$q(Z,T) = -\eta N_2^{-1/2}(0) \exp(\ \frac{C}{2}Z \ ) \text{ sech } [\eta T \exp(CZ)] \tag{25}$$

$$\exp\left[-iT^2\frac{C}{2}\exp(CZ) - i\frac{1}{2}\eta^2 Z \exp(CZ)\right] \tag{26}$$

$$q(Z,T) = \eta N_2^{-1/2}(0) \exp(\ \frac{C}{2}Z \ ) \text{ th } [\eta T \exp(CZ] \tag{27}$$

$$\exp\left[iT^2\frac{C}{2}\exp(CZ) - i\eta^2 Z \exp(CZ)\right] \tag{28}$$

Here T and Z are ordinary variables and C is arbitrary constant. If we use the expressions D_2(Z)=constant and N_2=N_2(0) then we obtain the following solutions of Eq. (3) in the form of hyperbolically growing ideal bright and dark solitons (for the first time reported in [12,13]

$$q(Z,T) = -\frac{\chi \ N_2^{-1/2}(0)}{(1 - 2\Gamma(0)Z)} \text{ sech}\left[\frac{\chi T}{(1 - 2\Gamma(0)Z)}\right] \ \exp\left[-i\frac{T^2\Gamma(0)}{(1 - 2\Gamma(0)Z)} - i\frac{\chi^2 Z}{2(1 - 2\Gamma(0)Z)}\right] \tag{29}$$

$$q(Z,T) = \frac{\chi \ N_2^{-1/2}(0)}{(1 - 2\Gamma(0)Z)} \text{ th}\left[\frac{\chi T}{(1 - 2\Gamma(0)Z)}\right] \exp\left[i\frac{T^2\Gamma(0)}{(1 - 2\Gamma(0)Z)} - i\frac{\chi^2 Z}{(1 - 2\Gamma(0)Z)}\right] \tag{30}$$

In the case of $\Gamma(Z) \equiv G_0$ and N_2=$N_2(0)$ the solution of Eq. 3 is given by:

$$Q(P(Z)T) = \eta N_2^{-1/2}(0) \text{ sech } [\eta P(Z)T] \tag{31}$$

$$Q(P(Z)T) = \eta N_2^{-1/2}(0) \text{ th } [\eta P(Z)T] \tag{32}$$

$$P(Z) = -P(0)\exp(\frac{1}{2G_0}(\exp(2G_0 Z) - 1)) \tag{33}$$

$$D_2(Z) = D_2(0)\exp(2G_0 Z - \frac{1}{2G_0}(\exp(2G_0 Z) - 1)) \tag{34}$$

When GVD is a hyperbolically decreasing function of Z

$$D_2(Z) = \frac{1}{1 + \beta Z} \tag{35}$$

then from Lemma 1 follows the explicit soliton solution in the form of Eq. 4

$$P(Z) = -\frac{1}{1 - \frac{1}{\beta}\ln(1 + \beta Z)} \tag{36}$$

$$\Gamma(Z)=\frac{1}{2(1+\beta Z)}\left[\frac{1-\ln(1+\beta Z)}{1-\frac{1}{\beta}\ln(1+\beta Z)}\right] \tag{37}$$

Let us consider the soliton intensity management problem. Chirped soliton pulse of Eq. 3 with the constant intensity can be obtained by using Lemma 2

$$P(Z)=-CZ-1;\ D_2(Z)=C/(1+CZ)^2;\ \Gamma(Z)=-C/2/(1+CZ) \tag{38}$$

Let us consider some periodical chirped soliton solutions of Eq. 3. Suppose that the soliton intensity varies periodically as

$$D_2(Z)P^2(Z)=\Theta(Z)=1+\delta\sin^{2n}Z \tag{39}$$

Then soliton solution in the case of n=2 is determined by Eq. 4 with parameters:

$$D_2(Z)=\Theta(Z)/P^2(Z);\ P(Z)=C-\left[Z+\delta\left(\frac{3Z}{8}-\frac{\sin 2Z}{4}+\frac{\sin 4Z}{32}\right)\right] \tag{40}$$

$$\Gamma(Z)=\frac{1}{2}\frac{\left(1+\delta\sin^4 Z\right)}{C-\left[Z+\delta\left(\frac{3Z}{8}-\frac{\sin 2Z}{4}+\frac{\sin 4Z}{32}\right)\right]}+\frac{1}{2}\frac{2\ \sin 2Z\ \sin^2 Z}{\left(1+\delta\sin^4 Z\right)} \tag{41}$$

Let us consider some periodical solutions of Eq. 3 in the case of the linearly dependent parameters of the media. The simplest solution of Eq. 3 in the form of Eq. 6 is:

$$P(Z)=\Upsilon(Z)=-\left(1+\delta\sin^2 Z\right);\ N_2(Z)=D_2(Z)=\frac{\delta\sin 2Z}{\left(1+\delta\sin^2 Z\right)^2}; \tag{42}$$

$$\Gamma(Z)=\frac{\delta}{2}\frac{\sin 2Z}{\left(1+\delta\sin^2 Z\right)} \tag{43}$$

The next periodical soliton solution is given by

$$D_2(Z)=N_2(Z)=\cos Z;\ P(Z)=-\frac{1}{(C-\sin Z)};\ \Gamma(Z)=\frac{\cos Z}{2\,(C-\sin Z)} \tag{44}$$

The main soliton features of the solutions given by theorem 1 and theorem 2 were investigated by using direct computer simulations. We have investigated the interaction dynamics of particle-like solutions obtained, their soliton-like character was calculated with the accuracy as high as 10^{-9}. We also have investigated the influence of high-order effects on the dynamics of dispersion and amplification management. As follows from numerical investigations elastic character of chirped solitons interacting does not depend on a number of interacting solitons and their phases. Figure 2 shows the computer simulation dynamics of three hyperbolically growing solitons Eq. 29. NSE solution with periodic dispersion coefficient is shown in Figure 3. Here the dispersion management function is

$$D_2(Z)=1+\delta\sin^2(Z) \tag{45}$$

and the soliton solution is given by Eqs. 17-18. In Figure 3 parameters C=200 and $\delta=-0.9$. Figure 4 represents the two dispersion managed solitons interaction in the case of equal phases and in Figure 5 the interaction dynamics of two solitons is shown in the case of opposite phases. Figure 6 shows the intensity managed solitons dynamics of the form presented by Eq. 38. Figures 7-9 show the nonlinear propagation and interaction of the dispersion and nonlinear managed solitons of Eqs. 42-43. The main parameters in computer simulations were C=200; $\delta=\pm 0.9$. Figure 10 illustrates the dynamics of the fission of the bound states of two hyperbolically growing solitons Eqs. 29-30 produced by self-induced Raman scattering effect given by Eqs.2-3. This remarkable fact also emphasize the full soliton features of solutions discussed. They not only interact elastically but they can form the bound states and these bound states split under perturbations. The possibility to find the plethora of soliton solutions in the case of strong dispersion management is reported in the recent paper of Zakharov and Manakov [14].

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FIG. 1. Femtosecond soliton spectral trapping effect.

FIG. 2. Mutual interaction of three hyperbolically growing chirped solitons of Eq. 29 in the case of equal amplitudes and phases.

FIG. 3. Evolution of the chirped dispersion managed solitary wave of Eqs. 17 and 18 as a function of the propagation distance. Dispersion managed function is periodic of the form Eq. 45. Input conditions : C=200 and δ=-0.9.

FIG. 4. Two dispersion managed solitons of Eqs. 17-18 and 45 interaction for the case of equal phases. Input conditions: C=200 and δ=-0.9.

FIG. 5. Two dispersion managed solitons of Eqs. 17-18 and 45 interaction for the case of equal phases. Input conditions: C=200 and δ=-0.9.

FIG. 6. Two intensity managed solitons of Eq. 38 interaction for the case of zero initial group velocities.

FIG. 7. Evolution of the chirped solitaty wave of Eqs. 39-41 as a function of the propagation distance. Input conditions: δ=0.9 and group velocity V_0=10.

FIG. 8. Evolution of the chirped solitary wave of Eqs. 39-41 for the case: δ =-0.8 and group velocity V_0=2.0.

FIG. 9. Soliton dispersion trapping effect in the presence of the linearly dependence between the nonlinearity and GVD parameters.

FIG. 10. Decay of high-order hyperbolically growing solitons in the presence of third-order dispersion and Raman self-scattering effects.